S.6 APPLIED MATHEMATICS  
TEST 3, TERM 2 2019  
NUMERICAL METHODS  
Time: 1 hour 30 minutes

Attempt all questions.

1. Given below are the values of \( f(x) \) for corresponding value of \( x \).  
\[
    f(0.4) = -0.9613, \quad f(0.6) = -0.5103 \quad \text{and} \quad f(0.8) = -0.2231
\]
Use linear interpolation to determine;
   i) \( f(0.7) \) correct to 4 decimal places, \hspace{1cm} (03 marks)
   ii) \( f^{-1}(-0.4308) \) correct to 2 decimal places. \hspace{1cm} (02 marks)

2. Given that \( y = \sin \theta \) and \( \theta \) is measured with a maximum possible error of 2%. If \( \theta = 30^\circ \) determine the;
   (i) absolute error in \( y \), \hspace{1cm} (02 marks)
   (ii) interval with in which the value of \( y \) lies. Give your answer correct to 4 significant figures. \hspace{1cm} (03 marks)

3. Study the flowchart below and answer the questions that follow.

   (i) Perform a dry run for the above flow chart. \hspace{1cm} (05 marks)
   (ii) Suggest a purpose of the flow chart.
4. a) Use the trapezium rule with six ordinates to find the approximate value of 
\[ \int_2^5 xe^{-x} \, dx \] correct to three significant figures.
b) Find the area bounded by the curve \( y = xe^{-x} \) between \( x = 2 \) and \( x = 5 \).
c) Find the percentage error in (a) above.

(12 marks)

5. The numbers \( A \) and \( B \) are rounded off to \( a \) and \( b \) with errors \( e_1 \) and \( e_2 \) respectively.

a) Show that the absolute relative error in the product \( AB \) is given by:
\[ \frac{|a||e_2|+|b||e_1|}{ab} \]  
(05 marks)
b) Given that \( A = 6.43 \) and \( B = 37.2 \) are rounded off to the given number of decimal places indicated;
   i) State the maximum possible errors in \( A \) and \( B \).  
   (02 marks)
   ii) Determine the absolute error in \( AB \).  
   (02 marks)
   iii) Find the limits within which the product \( AB \) lies. Give your answer to 4 decimal places.  
   (03 marks)

6. Given the equation; \( x^3 - 6x^2 + 9x + 2 = 0 \).
   a) Find graphically the root of the equation which lies between -1 and 0.  
   (05 marks)
   b) i) Show that the Newton Raphson formula for approximating the root of the equation is given by \( x_{n+1} = \frac{2}{3} \left( \frac{x_n^3 - 3x_n^2 - 1}{x_n^2 - 4x_n + 3} \right) \) where \( n = 0, 1, 2, \ldots \)  
   (03 marks)
   ii) Use the formula in b(i) above, with an initial approximation in a) above to find the root of the given equation correct to two decimal places.  
   (04 marks)

END