PRIMARY SIX
SELF STUDY MATERIALS
ENGLISH LANGUAGE
PRIMARY SIX
LESSON 1: Use of Adverbs

Learning Outcomes
By the end of this lesson, you should be able to:

i) identify ways of doing things.

ii) use of adverbs in your daily life activities.

You will need:
• a pen
• a book

Introduction
An adverb is a word that tells more about a verb, an adjective and another adverb. Adverbs tell us how an action is done: when? why? how? and where?

Examples
i) The cyclist is riding slowly.

ii) Children should cross the road carefully.

Activity 1
Use the correct form of the word in the brackets to complete the sentences below. Examples have been given to guide you.

1. Tino cleverly avoided the punishment. (clever)

2. Uganda's population is steadily rising. (steady)

3. St Thomas choir sang the second compulsory song beautifully. (beauty)

4. The host welcomed us cheerfully. (cheer)

5. We patiently waited for his return from Ojipaku market. (patience)

6. Italians have seen the worse coronavirus attacked. (worse)

7. They answered the question wisely. (wise)

8. A machete is usually bigger than a knife. (usual)

9. The hungry boy ate the mango greedily. (greed)

10. The hunter bravely attacked the lion in its den. (brave)

Activity 2
Order of Adverbs
When more than one adverb is used in a sentence, they usually follow in this order: manner (how?), place (where?) and time (when?).

Example
The pedestrian crossed the road carefully (how?) at the zebra crossing (where?) yesterday (when?).

From the sentences below, underline the adverbs of manner, place and time.

1. The traffic officer called the driver loudly at the junction in the morning.

2. Many cyclists ride carelessly at the traffic lights every day.

3. The bus driver spoke politely to the passenger on the bus on Tuesday.

4. My mother nicely made cakes from the kitchen last weekend.

5. It rained heavily in our area last year.
1. (a) A purchase order form has the following contents:
- Date
- Address of the supplier (name & address)
- Inside address
- Document title “purchase order”
- Purchase order number
- Breakdown of machinery being ordered for: number,
- Description, quantity, unit cost, amount and grand total
- Delivery date
- Suggested type of packaging

   - Person making order
   - Frame
   - Level of completion or ease entry into the market
   - Availability of required raw materials

(b) Contents of the plant layout
- Access road with an arrow
- Reception
- Parking yard
- Administration block
- Production block
- Workers facilities
- Sanitary place
- Stores
- Water point
- Power house
- Waste disposal
- Frame

(c) A marketing plan
- Name and address with statement “marketing plan”
- Target customers (mentioned)

Pricing structure
- Products to be offered; i.e., maize flour and quantities
- Position of competitors (mentioned and their location)
- Pricing and sales strategy; i.e.,

(d) An effective advert has the following contents
- Name of the advertising medium and signpost
- Name and address with postal and telephone of the business
- Products being offered
- Benefits of the products to the customer
- An appealing phrase/message
- Arrow showing the direction of the business with (km)
- Stands
  - Note: Avoid congesting the advert

(d) A letter to the district engineer seeking technical advice on the proposed expansion

Contents of the letter
- Name and address of the business reflecting the project that is being produced
- Date
- Reference
- Inside address
- Salutation; i.e, Dear sir

(b) Contents of the financial plan
(i) Name and address of the business with statement “financial plan”

Estimated fixed capital requirements of the business

<table>
<thead>
<tr>
<th>ITEM</th>
<th>AMOUNTS (SHS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed capital expenses</td>
<td></td>
</tr>
<tr>
<td>Acquisition of land</td>
<td>XX</td>
</tr>
<tr>
<td>Construction of building</td>
<td>XX</td>
</tr>
<tr>
<td>Purchase of machinery and equipment</td>
<td>XX</td>
</tr>
<tr>
<td>Purchase of tools</td>
<td>XX</td>
</tr>
<tr>
<td>TOTAL FIXED CAPITAL</td>
<td>XXX</td>
</tr>
</tbody>
</table>

(ii) Estimated working capital requirement

<table>
<thead>
<tr>
<th>Working capital expenses</th>
<th>AMOUNTS (SHS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Raw materials</td>
<td>XX</td>
</tr>
<tr>
<td>Power and utilities</td>
<td>XX</td>
</tr>
<tr>
<td>Packaging materials</td>
<td>XX</td>
</tr>
<tr>
<td>Direct labour</td>
<td>XX</td>
</tr>
<tr>
<td>Administrative expenses</td>
<td>XX</td>
</tr>
<tr>
<td>Marketing expenses</td>
<td>XX</td>
</tr>
<tr>
<td>TOTAL</td>
<td>XXX</td>
</tr>
</tbody>
</table>

(iii) Expected sources of funds.

<table>
<thead>
<tr>
<th>SOURCE OF FUNDS</th>
<th>AMOUNT (SHS)</th>
<th>COST IMPLICATIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Own funds</td>
<td>XX</td>
<td></td>
</tr>
<tr>
<td>Family and friends</td>
<td>XX</td>
<td></td>
</tr>
<tr>
<td>Trade credit</td>
<td>XX</td>
<td></td>
</tr>
<tr>
<td>Loan</td>
<td>XX</td>
<td></td>
</tr>
<tr>
<td>TOTAL</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(IV) Estimated sales per year shs XXX
(V) Expected profit of the business per year

<table>
<thead>
<tr>
<th>DETAILS</th>
<th>AMOUNT (SHS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales</td>
<td>XXX</td>
</tr>
<tr>
<td>Less: Estimated products</td>
<td>XX</td>
</tr>
<tr>
<td>Expenses</td>
<td>XX</td>
</tr>
<tr>
<td>Gross profit</td>
<td>XX</td>
</tr>
<tr>
<td>Less: administrative over needs</td>
<td>XX</td>
</tr>
<tr>
<td>Tax to be</td>
<td>XX</td>
</tr>
<tr>
<td>Next profit</td>
<td>XX</td>
</tr>
</tbody>
</table>

(VI) Cash flow projections of the business: i.e.,
- Cash inflow – Cash outflow = Net cash

(VII) Break-even scale of the business: the expected break-even scale of the business is where total cost is equal to total revenue

(c) Operational budget to finance the expansion of the cooking oil business.

- Name and address with the word operational budget.

Organisation chart

- Board of directors
  - Managing director
    - General manager
    - Production Manager
    - Finance Manager
    - Marketing Manager

Staff

Turn to next page
3. You plan to start a business based on the environment in your community.
(a) Factors considered when formulating the business idea other than market include:
- Return on the investment/profitability
- Level of completion or ease entry into the market
- Availability of required raw materials
- Availability of market
- Cost of the business venture
- Availability of required raw materials
- Availability of labour
- Availability of required technology
- Availability of land
- Strategic business location
- Business risks involved
- Availability of infrastructure and other services
- Personal consideration or interest
- Acceptability in the community
- Government policy or support to the business
- Nature of the customers

(b) Personal entrepreneurial skills to make your business
- Planning skills
- Leadership
- Marketing skills
- Organisational skills/co-ordinating skills
- Time management skills
- Communication skills
- Controlling skills
- Budgeting skills
- Decision making skills
- Negotiating skills
- Purchasing skills
- Interpersonal skills
- Technical skills
- Net working
- Innovation and creativity skills
- Coping with change skills

(c) Social concerns to be met by the business
- Unemployment
- Poverty
- Natural disasters like floods and epidemics like coronavirus (COVID-19)
- Environment degradation
- School dropout
- Drug abuse
- Unsafe water and lack of safe water
- Inadequate health education
- Disadvantages groups like women, people with disabilities and orphans
- Inadequate supply of goods and services
- Insecurity both political and social
- Bad foreign influence; e.g., gayism/moral decay
- Domestic violence
- Inadequate market for products

(d) Aspects to be addressed during business implementation planning.
It involves using the contents of the action plan which include:
- Resources needed in implementation of the planned activities
- Time frame; i.e., how long each activity is to take
- Persons in charge – workers responsibilities
- Indicators of success
- Remarks – regards to work done
- Activities to be carried out

4. (a) BULA ENTERPRISES LTD’S CASH FLOW STATEMENT FOR THE MONTHS JAN – APRIL 2019

<table>
<thead>
<tr>
<th>DETAILS</th>
<th>JAN</th>
<th>FEB</th>
<th>MARCH</th>
<th>APRIL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash in flow</td>
<td>shs</td>
<td>shs</td>
<td>shs</td>
<td>shs</td>
</tr>
<tr>
<td>Opening balance</td>
<td>20,000,000</td>
<td>(2,200,000)</td>
<td>(2,664,000)</td>
<td>8,219,450</td>
</tr>
<tr>
<td>Net sales</td>
<td>8,000,000</td>
<td>8,400,000</td>
<td>8,820,000</td>
<td>9,261,000</td>
</tr>
<tr>
<td>Loan principal repayment</td>
<td>5,500,000</td>
<td>5,500,000</td>
<td>5,500,000</td>
<td>5,500,000</td>
</tr>
<tr>
<td>Loan interest payment</td>
<td></td>
<td></td>
<td>10,000,000</td>
<td></td>
</tr>
<tr>
<td>Sale of old vehicle</td>
<td>335,500,000</td>
<td>11,700,000</td>
<td>(3,320,000)</td>
<td>57,433,500</td>
</tr>
<tr>
<td>Total cash inflow</td>
<td>361,000,000</td>
<td>11,700,000</td>
<td>(3,320,000)</td>
<td>57,433,500</td>
</tr>
<tr>
<td>Less: Cash outflow</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Purchasing of machine</td>
<td>9,750,000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Import duty</td>
<td></td>
<td>487,500</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Salaries &amp; wages</td>
<td>30,000,000</td>
<td>30,000,000</td>
<td>60,000,000</td>
<td></td>
</tr>
<tr>
<td>Average stock</td>
<td>500,000</td>
<td>1,000,000</td>
<td>1,000,000</td>
<td>1,000,000</td>
</tr>
<tr>
<td>Loan interest payment</td>
<td>20,000</td>
<td>20,000</td>
<td>20,000</td>
<td>20,000</td>
</tr>
<tr>
<td>Purchase of raw materials</td>
<td>5,400,000</td>
<td>5,940,000</td>
<td>6,237,000</td>
<td>6,548,850</td>
</tr>
<tr>
<td>Total cash outflow</td>
<td>35,700,000</td>
<td>3,834,000</td>
<td>78,874,500</td>
<td>89,488,850</td>
</tr>
<tr>
<td>Net cash position</td>
<td>2,200,000</td>
<td>2,664,000</td>
<td>82,194,500</td>
<td>66,382,350</td>
</tr>
</tbody>
</table>

(b) Personal entrepreneurial skills to make your business
- Formulating the business idea other community.
- Other services
- Materials
- Profitability
- More than market include;
  - Formulating the business idea other community.
  - Other services
  - Materials
  - Profitability

5. (a) Gross profit margin = profit margin ×100
   Gross sales
   (b) Cost of sales
   Net sales
   30%
   (c) Stock turnover = cost of sales
   Average stock
   Net sales - (opening stock + closing stock)/2
   45,300,000
   - 360,000,000
   - 4,5 times
   (d) Debtors collection period = number of days in a year
   - 366
   - 114 days
   (e) Leverage ratio / gearing ratio = long-term liabilities
   Owner’s equity
   1:0.56 or (56.3%)
CHEMISTRY PAPER 1 SET 3 ANSWERS

1. a) Pt(s) / I- (aq) // Fe3+(aq), Fe2+(aq)/pt(s)
   b) i) 2I-(aq) → I2(aq) + 2e–
   Fe3+(aq) + e– → Fe2+(aq)
   ii) 2Fe3+(aq) + 2I–(aq) → 2Fe2+(aq) + I2(aq)
   c) i) $E_{cell} = E_{cell}^{0} - E_{0}$
   $E_{cell}^{0} = 0.76$ – 0.34
   $E_{0} = +0.22$V
   ii) Feasible, because the e.m.f. of the cell is positive.

2. a) $C_{x}H_{y}$ + (x + $\frac{y}{4}$)O2 → CO2 + y/2 H2O
   b) Ca(s) $\rightarrow$ Ca2+ + 2e–
   H2/lindlard → H2C = CH2

3. a) $C_{6}H_{5}COO–(aq) + H_{2}O(l) \rightarrow C_{6}H_{5}COOH(aq) + OH(aq)$
   b) $K_{h} = \frac{[C_{6}H_{5}COOH][OH]}{[C_{6}H_{5}COO–]}$
   $K_{h} = 1.2 \times 10^{-2}$

4. a) Fe3+(aq) + 3H2O(l) → Fe(OH)3(s) + 3H+(aq)
   Fe(H2O)63+(aq) + 3H+(aq) → Fe(H2O)3(OH)3(s) + 3H+(aq)
   Fe(H2O)63+(aq) + H2O(l) → [Fe(H2O)5OH]2+ + H3O+(aq)
   b) i) brown precipitate and effervescence/bubbles of a colourless gas
   ii) 2Fe3+(aq) + 6H2O(l) → 2Fe(OH)3(s) + 3H2(g)
   2Fe(H2O)63+ + 3Mg(s) → 2Fe(H2O)3(OH)3(s) + 3Mg2+(aq) + 3H2(g)

5. a) NO2
   b) $N\equiv N$ > $N\equiv O$

6. a) 2F2(g) + 2H2O(l) → 4HF(aq) + O2(g)
   b) 2F2(g) + 2OH(aq) → OF2(g) + 2F–(aq) + H2O(l)
   c) 2F2(g) + 4OH(aq) → 4F–(aq) + 2H2O(l) + O2(g)

7. a) Remainder of $10 - 5.56 = 4.4g$
   let w g be extracted
   b) $w = \frac{4.4}{50} \times 100 = 0.2$ w/50
   $w = 2.47g$
   Total mass extracted = 5.56 + 2.47 = 8.03g

8. a) | Reagent | Reaction |
   --- | --- |
   CH3CH2CH2CH2CH2OH | Anhydrous zinc chloride (VII) solution |
   CH3COONa | Acetate ion solution |
   CH3CONa | Sodium acetate solution |
   Fe3+(aq) | Iron (III) chloride solution |

9. a) | Shape | Structure |
   --- | --- |
   Square pyramidal | Fe(II) 6 coordination |
   Trigonal pyramidal | Fe(III) 5 coordination |

10. a) CH1–CH2–OH + NaOH(aq) → CH3CH2ONa + H2O
    b) $\text{Sn} > \text{conc HCl}$

Turn to next page
11. a) i) write the electronic configuration of chromium atom

15S^25s^25p^63d^54s^1 or 15s^25p^63d^54s^1

ii) Because it has partially filled d-orbitals

b) [Cr(H2O)6]^3+ , [Cr(H2O)5Cl]^2+ , [Cr(H2O)4Cl2]^+ , Cr(H2O)3Cl3 , Cr(H2O)3Cl2 , CrCl4

c) i) Grey green ppt, soluble in excess giving a purple or violet solution

ii) Cr^2+(aq) + 3OH(aq) \rightarrow Cr(OH)3(s) or Cr(OH)4^- (aq) + 3OH(aq)

12. Mass of carbon = \frac{12}{14} \times 17.6 = 14.8g

Mass of oxygen = 6.8 \times (4.8+0.4) = 1.6g

C H O

8.4 0.4 1.6

12 0.4 1.6

0.4 0.4 0.1

0.1 0.4 0.1

4 4 1

Empirical is C4H4O

b) (C4H4O)n = 68 \times 2 = 136

C H O

12 1 6

Molecular formula = C8H8O2

13. a) is the precipitation of a sparingly soluble electrolyte from its saturated solution by adding a solution of amore soluble electrolyte containing one of its ions.

b) i) Ag2CrO4(s) + 2H+(aq) \rightarrow 2Ag+(aq) + CrO4^{2-}(aq)

c) i) ClO3^(-)(aq) + H+ + H2O(l) \rightarrow ClO4^-(aq) + 2H2O(l)

ii) Addition polymerisation

iii) \frac{v = nRT}{\pi} = \frac{5 x 8.31 x 298}{4.0 \times 10^{-3} \times 101325 x 10^{-3}} = 152749.8

iv) Rfn of CH2=CH–C = CH2 is 88.5

v) Volume of HCl added/cm^3

(vi) Which of the indicators shown below is suitable for the titration? Give a reason for your answer.

CH3CH=CH2

CH3CHCH3

(b) Use the graph to determine the;

- Volume of HCl added/cm^3
- pH and volume at the end point

2. The table below shows the variation in pH when 30cm^3 of 0.2M ammonia solution was titrated with hydrochloric acid:

<table>
<thead>
<tr>
<th>Volume of HCl added/cm^3</th>
<th>0</th>
<th>4</th>
<th>8</th>
<th>12</th>
<th>16</th>
<th>18</th>
<th>19</th>
</tr>
</thead>
<tbody>
<tr>
<td>pH</td>
<td>10.8</td>
<td>9.9</td>
<td>9.4</td>
<td>9.1</td>
<td>8.7</td>
<td>8.3</td>
<td>8.0</td>
</tr>
</tbody>
</table>

(a) Plot a graph of pH against volume of HCl added

(b) Use the graph to determine the;

- pH and volume at the end point
- Molarity of hydrochloric acid solution

(c) Hydrolysis constant of ammonium chloride formed at the end point

(iv) Ratio of [NH4Cl]: [NH3] when 10cm^3 of HCl has been added to ammonia solution

(K for ammonia \approx 1.78 \times 10^{-5}mol\cdotdm^{-3}, K_b = 1 \times 10^{-10}mol\cdotdm^{-3})

(c) Explain the shape of the graph

(d) Which of the indicators shown below is suitable for the titration? Give a reason for your answer.
3. When 7.5g of an organic compound Q was burnt completely in excess oxygen 11.2dm³ of carbon dioxide and 4.5g of water were formed at 1atm. 
   (a) (i) Calculate the empirical formulat of Q 
   (ii) Determine the molecular formulat of Q (Density of Q is 3.537g/ml at 1atm)
   (iii) Q burns with a sooty flame and forms a yellow precipitate with 2,4-dinitrophenylhydrazine and also forms a pale yellow precipitate with iodine solution in sodium hydroxide solution. Identify Q 
(b) Write equation and suggest a mechanism for the reaction between Q and: 
   (i) 2,4-dinitrophenylhydrazine in acidic medium 
   (ii) Sodiumhydrogen sulphate solution
(c) Using equations only show how Q: 
   (i) Can be synthesized from benzaldehyde 
   (ii) Can be converted to an alkene
4(a) (i) State Le Chaterier’s principle 
   (ii) State two factors that affect equilibrium reactions apart from catalyst 
(iii) Briefly describe how each of the factors you have named in (a)(i) affect the equilibrium constant and equilibrium position. 
(b) Given the reaction; 
   \[ \text{Ni(CO)}_4(\text{g}) \rightarrow \text{Ni}^{(s)} + 4\text{CO}^{(g)} \]
   Write the expression for the equilibrium constant \(K_c\) and \(K_p\) giving units in each case.

**CHEMISTRY PAPER 2 QUESTIONS (ACHEM04)**

What is the effect on equilibrium position of adding a catalyst 
(c) COCl₃ dissociates according to the following equation; 
   \[ \text{COCl}_3(\text{g}) \rightleftharpoons \text{CO}_2(\text{g}) + \text{Cl}_2(\text{g}) \]
   (i) At 25°C, one mole of COCl₃ was placed in 2dm³ vessel producing an equilibrium mixture with 20.25% chlorine. Calculate the value of the equilibrium constant \(K_c\) at this temperature.
   (ii) At 75°C, the degree of dissociation of 2 moles of COCl₃ in the same 2dm³ vessel was found to be 15%. Calculate the value of the equilibrium constant \(K_c\) at this temperature.
   (iii) From your answer in (i) and (ii) above, state whether the reaction is exothermic. Give a reason for your answer.

**SECTION B**

5. Write equation to show how the following conversion can be affected. 
   (a) Bromo benzene from phenol 
   (b) Propanol from chloroethane 
   (c) Phenyl amine from methylbenzene 
   (d) Hexane from propane 
   (e) (CH₂)₃=NC=CH₂,CH₃, from 2-iodopropane
6. (a) Define the following terms 
   (i) Latent energy 
   (ii) Hydration energy 
   (b) Given the following data; Standard enthalpy of formation of aluminium fluoride = -1301 KJ/mole 
   Enthalpy of atomization of aluminium = 314 KJ/mole
   Bond dissociation energy of fluorine = 158 KJ/mole 
   First ionization energy of aluminium = 577 KJ/mole 
   Second ionization energy of aluminium = 1820 KJ/mole 
   Third ionization energy of aluminium = 2740 KJ/mole 
   First electron affinity of fluorine = -348 KJ/mole 
   (i) Draw an energy level diagram for the formation of aluminium fluoride and use it to determine the lattice energy of aluminium fluoride
   (ii) If the hydration energies of aluminium ions and fluoride ions are -4690 and -364 KJ/mole respectively, calculate the enthalpy of solution of aluminium fluoride and hence comment on the solubility of aluminium fluoride 
   (iii) State and explain two factors that affect lattice energy. 
   (e) Would you expect lattice energy of aluminium chloride to be less than, equal to, or greater than that of aluminium fluoride? Explain your answer.
7. (a) Describe the industrial preparation of chlorine. 
(b) Describe the reaction of chlorine with 
   (i) Iron 
   (ii) Potassium hydroxide 
(c) Describe the reaction of fluorne with: 
   (i) Water 
   (ii) Potassium hydroxide 
   (d) Explain why hydrogen fluoride is a liquid whereas hydrogen chloride is a gas at 18°C.
8. (a) Explain the following processes as used in the extraction of metals: 
   (i) Floatation 
   (ii) Roasting 
   (iii) Smelting 
   (b) Briefly describe how the ore of aluminium can be concentrated.
11. A collided and coalesced with a stationery paper patients.

(ii) Show that the maximum possible relative error in volume is \( \frac{\Delta V}{V} = \frac{\Delta h}{h} + \frac{\Delta r}{r} \).

(iii) If \( h \) and \( r \) are measured with percentage errors of 2% and 1.4% respectively, find the corresponding percentage error in volume.

14. a) A random sample of 36 chicken from a poultry farm is taken. These have mass (measured in kg) of \( x \), such that
\[ \sum x = 144.09 \text{ and } \sum x^2 = 588.69. \]
Calculate a 95% confidence interval for the mean.

b) The weights of ball bearing are normally distinguished with mean 25g and standard deviation 4grams. If a random sample of 16 ball bearings is taken, find the probability that the mean of the sample lies between 24.12 grams and 26.73 grams.

15. a) In S6 class, 40% of the candidates support David for the post of head prefect. If a random sample of 10 candidates is selected, find the probability that more than 5 candidates support David for the post.

b) An examination has two parts A and B, the probability of a student getting part A correct is \( \frac{1}{2} \). If she gets part A correct, the probability that she gets B correct is \( \frac{1}{3} \). Otherwise it is \( \frac{1}{6} \). There are three marks for a correct solution of part A, two marks for a correct solution of part B and a bonus mark if both parts are correct.

(i) Construct a probability distribution table for the total marks scored by the student.

(ii) Find the expected student’s total mark.

16. a) Show that the centre of mass of a uniform semi-circular laminar is \( \frac{4r}{3\pi} \) from the bounding diameter.

b) The figure below shows a rectangular piece of cardboard ABCD measuring 30cm by 12cm.

If a semi-circle of radius 5cm is removed:

(i) Calculate the coordinates of the centre of mass of the remaining piece of cardboard taking BC and AB as the X and Y axes respectively.

(ii) Given that the remaining card is suspended from A and hangs in equilibrium, find the angle AD makes with the vertical.

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**SECTION A**

1. \( 3 \cos^2 x - 3 \sin x \cos x + 2 \sin^2 x = 1 \)

Dividing through by \( \cos x \),
\[ 3 - 3 \sin x + 2 \tan x - 1 + \tan^2 x \]
\[ \tan^2 x - 3 \tan x + 2 = 0 \]
\[ \tan(x - 1) - 2 \tan(x - 1) = 0 \]
\[ (\tan(x - 1) - 1)(\tan(x - 1) - 2) = 0 \]
\[ \tan x = 1 \implies x = \tan^{-1} 1 \]
\[ x = 45^\circ, 225^\circ \]

Therefore, \( x = 63.4^\circ, 243.4^\circ, 45^\circ \) and \( 225^\circ \).

2. Consider L.H.S using \( \int \frac{dx}{x} \)
\[ \int \log_2 10^x = \int 2^x \]
\[ \int \log_2 \frac{dx}{10^x} = \int \frac{dx}{x} \]
\[ \frac{1}{2} \log_2 \frac{x^2}{2} - \frac{1}{2} \ln 10 \]
\[ = \frac{1}{2} \left( \log_2 x^2 - \ln 10 \right) \]
\[ = \frac{1}{2} \left( \log_2 x^2 - \ln 10 \right) \]
\[ = 10 - x \sin 10 - \ln x \]
\[ = x^2 + c \]

3. \( \frac{z - 1}{z + 1} = 2 \)

Let \( z = x + iy \)
\[ x + iy = \frac{2(2x + iy)}{x + iy} \]
\[ x + iy = 2 \left( \frac{x - y}{x + y} \right) \]
\[ x^2 - y^2 + 2x + 1 = 2 \left( \frac{x - y}{x + y} \right) \]
\[ x^2 + y^2 + 2x + 1 = 4x^2 + 2y^2 \]
\[ 3x^2 + 3y^2 + 10x + 3 = 0 \]
\[ x^2 + y^2 + 1 = 0 \]

The locus is a circle centre \((1,0)\) and radius \( \frac{\sqrt{10}}{3} \).

4. \( a + b = 26, a^2 + b^2 = 10 \)

Sum of the roots:
\[ a + b = 26 \]
\[ a^2 + b^2 = 10 \]
\[ (a + b)^2 = 26^2 \]
\[ = 676 \]
\[ a^2 + b^2 = 10 \]
\[ a^2 + b^2 = 100 \]
\[ a^2 + b^2 = 10 \]
\[ x^2 + y^2 = 26 + 10 = 36 \]
\[ x^2 + y^2 = 26 \]
\[ x^2 + y^2 = 10 \]

5. \( y = \ln \tan \alpha \)
\[ \frac{dy}{d \theta} = \sec^2 \alpha \cdot \tan \alpha \]
\[ = \frac{1}{2} \tan \alpha \]
\[ = \frac{1}{2} \tan \alpha \]
\[ \frac{dy}{d \theta} = \cos \alpha \cdot \tan \alpha \]
\[ = \cos \alpha \cdot \tan \alpha \]
\[ d^2 y \frac{dy}{d \theta} = -\cos \alpha \cdot \cot \theta \]

6. \( d_1 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \)
\[ d_2 = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} \]
\[ \cos \theta = \frac{d_1 \cdot d_2}{|d_1| |d_2|} \]
\[ = \frac{1}{\sqrt{10}} \]
\[ = \frac{20}{\sqrt{100}} \]
\[ = \cos^{-1} \left( \frac{2}{5} \right) \]
\[ = 0.60 \]

7. Using the general equation of the circle \( x^2 + y^2 + 2gx + 2fy + c = 0 \)
At \((2, 3)\)
\[ 4 + 9 + 4g + 6f + c = 0 \]
\[ 4g + 6f + c = -13 \]

At \((3, 2)\)
\[ 9 + 4 + 6g + 4f + c = 0 \]
\[ 6g + 4f + c = -13 \]

At \((6, 1)\)
\[ 36 + 1 + 12g + 2f + c = 0 \]
\[ 12g + 2f + c = -37 \]

Eliminating \( c \) in equation (i) and (ii) gives \( 2g + 2f = 0 \)
\[ g = \frac{-2f}{2} = -f \]

Eliminating \( c \) in equation (i) and (iii) gives \( -8g + 4f = -24 \)
\[ -2g + f = 6 \]

Substituting equation (iv) in equation (v) gives \( -2g + g = 6 \)
\[ g = 6 \]

From (iv) \( f = -6 \)

Substituting \( f = -6 \) and \( g = -6 \) in equation (1) to get \( c \)
\[ -24 - 36 + c = -13 \]
\[ c = -13 + 60 \]
\[ c = 47 \]

Substituting \( g = -6, f = -6 \) and \( c = 47 \) to the general equation of the circle gives \( x^2 + y^2 = 12x - 12y + 47 = 0 \). Is the required equation of the circle.

8. \( \frac{dy}{dx} + 2y = e^{\alpha x} \)

If \( e^{\alpha x} \)
\[ e^{\alpha x} \]

Eqn (i) \( \cdot (ii) \) gives \[ e^{\alpha x} \frac{dy}{dx} + 2e^{\alpha x}y = e^{\alpha x} \]
\[ e^{\alpha x} \frac{dy}{dx} = e^{\alpha x} \]
\[ \int e^{\alpha x} \frac{dy}{dx} \ dx = \int e^{\alpha x} \ dx \]
\[ e^{\alpha x}y = \frac{1}{\alpha} e^{\alpha x} + c \]
for \( x = 0, y = \frac{5}{6} \) \( \frac{1}{2} \)
\[ c = 19 \]
\[ \frac{5}{6} + \frac{1}{2} + c \]
\[ c = 19 \]
\[ \frac{5}{6} + 19 \]
\[ \frac{30}{6} \]
**SOLUTIONS TO PURE MATHEMATICS PUBLISHED ON MAY 12**

**SECTION B**

9. a) Let \( P = \frac{3}{2} \)

\[
P = \frac{3 + 2}{3 - 2} = \frac{5}{1}
\]

\[
P = \frac{(3 + 2)(3 - 2) - 3}{3 - 2} = \frac{3}{1}
\]

\[
P = \frac{9 - (2i)^2}{3 - 2} = \frac{9}{1}
\]

\[
P = \frac{13 - 3}{3 - 2} = \frac{10}{1}
\]

Since \( i^2 = -1 \)

\[
P = \frac{30 + 20}{13} + \frac{20}{13}
\]

Therefore, \( Z = \frac{3}{2} \left( \frac{30}{13} + \frac{20}{13} \right) \)

\[
|P| = \sqrt{\left( \frac{30}{13} \right)^2 + \left( \frac{20}{13} \right)^2}
\]

\[
= \frac{\sqrt{1300}}{13}
\]

\[
= \frac{10}{13}
\]

Therefore \( \frac{2}{3} \left( \frac{30}{13} + \frac{20}{13} \right) \) (4dp’s)

Arg \( P = \tan^{-1} \left( \frac{1}{2} \right) \)

\[
= \tan^{-1} \left( \frac{1}{2} \right)
\]

= 33.7°

Therefore Arg \( \left( Z = \frac{3}{2} \right) \) 33.7° (1dp)

b) Let \( z = x + iy \)

\[
\frac{3(x + iy) + 1}{2(x + iy) - i} = \sqrt{2}
\]

\[
\frac{(3x + 1) + 3iy}{2x + (2y - 1)i} = \sqrt{2}
\]

\[
(3x + 1)^2 + 3y^2 = 2(2x + (2y - 1))^2
\]

\[
x^2 + 6x + 1 + 9y^2 = 2(4x^2 + 4y^2 - 4y + 1)
\]

\[
x^2 + 6x + 1 + 9y^2 - 8x^2 - 8y^2 - 8y - 2 = x^2 + y^2 + 6x + 8y - 1 = 0
\]

By completing squares

\[
(x + 3)^2 - 9 + (y + 4)^2 - 16 = 0
\]

\[
(x + 3)^2 + (y + 4)^2 = 26
\]

The focus is a circle centre (-3, -4) and radius \( r = \sqrt{26} \)

For \( x = 0 \)

\[
y = 4 + \frac{14}{5}x + 3c
\]

\[
C = \frac{2}{5}
\]

For \( x = 1 \)

\[
y = 8 + \frac{4}{5} + 4B + \frac{8}{5}
\]

\[
B = \frac{1}{5}
\]

\[
3x^2 + 4x + 4 = \frac{14}{5}(x + 3)^2 + \frac{14}{5}(x + 3) + 2
\]

Substituting equation (ii) in (i) gives

\[
\frac{x^2 + 1}{(x + 3)(x^2 + 1)} = 1 - \frac{14}{5(x + 3)} - \frac{2}{5(x + 3) - 5(x^2 + 1)}
\]

b) \[
\frac{x^2 + 1}{(x + 3)(x^2 + 1)} = \left\{ dx - \frac{1}{2} + \frac{1}{x^2 + 3} \right\} dx - \frac{2}{5(x + 3)} \frac{1}{5(x^2 + 1)} dx
\]

\[
x = \frac{\ln x + 3 - \frac{2}{5} \ln(x^2 + 1) - \frac{1}{2} \tan^{-1} x + c
\]

(a) Sketch the curve \( y = \frac{x^2 + 2}{(x - 2)(x + 2)} \) stating clearly the asymptotes.

**Solution**

\[
y = \frac{x^2 + 2}{x^2 - 4}
\]

\[
y = \frac{4x}{x^2 - 4}
\]

\[
(0, \frac{1}{2}) \text{ or } y > 1 \text{ from the table.}
\]

So \( y = \frac{1}{2} \) or \( y > 1 \)

For \( (y - 1)(4y + 2) \geq 0 \)

\[
y \leq \frac{1}{2} \text{ or } y \geq 1
\]

For \( y = 1 \)

\[
x^2 + 2 = 1
\]

\[
x = -1, 1
\]

\[
y = \frac{2}{x - 4}
\]

\[
2 - 4 \text{ (impossible) ignore it}
\]

Either \( y = \frac{1}{2} \) or \( y > 1 \)

\[
\frac{-1}{2} \leq \frac{x^2 + 1}{x - 4}
\]

\[
is the region.
\]

At \( y = 1 \), the turning point does not exist for \( y = \frac{1}{2} \)

the curve is maximum.

\[
\frac{-1}{2} \leq \frac{x^2 + 2}{x - 4}
\]

\[
2x^2 + 2 - x^2 + 4
\]

\[
x = 0
\]

\[
(0, \frac{1}{2}) \text{ is the maximum point}
\]

Vertical asymptotes \( x^2 - 4 = 0 \)

\[
x = \pm 2
\]

As \( x \to \pm \), for \( y = \frac{1}{2} \)

\[
y \to 1
\]

\[y = 1 \text{ is the horizontal asymptote.}
\]

**Critical values are \( x = \pm 2 \)**

\[
x^2 + 2 \quad 2x + 2 \quad x^2 - 4
\]

\[
y + + +
\]

\[
y - - +
\]

12. (a) i) \( \frac{x + 1}{x - 3} = \frac{z + 1}{z - 1} \)

\[
x = 2k - 1
\]

\[
y = 5k + 1 \to \text{ (I)}
\]

\[
z = -k - 1
\]

Substituting equation (I) to the equation of the plane \( 2x - y + 3z = 20 \) gives

\[
2(2k - 1) - (5k + 3) + 3(-k - 1) = 20
\]

\[
4k - 2 - 5k - 3 - 3k - 3 = 20
\]

\[
-4k = 28
\]

\[
k = -7
\]

Substituting \( k = -7 \) in equation (I) gives \( x = -15 \), \( y = -32 \) and \( z = 6 \)

Point of intersection is (-15, -32, 6)

(ii) Using \( \sin \theta = \frac{nd}{|OP|} \) where \( n = -\frac{2}{3} \)

\[
d = 5
\]

\[
\theta = \sin^{-1} \left( \frac{4}{\sqrt{20}} \right)
\]

\[
0 = -11.3^\circ
\]

(b) \[\frac{3}{4} \quad \frac{7}{4} \quad -\frac{7}{4} \quad -\frac{9}{4} \]

\[
\frac{3}{4} \quad \frac{7}{4} \quad -\frac{7}{4} \quad -\frac{9}{4}
\]

\[
\frac{3}{4} \quad \frac{7}{4} \quad -\frac{7}{4} \quad -\frac{9}{4}
\]

\[
\frac{3}{4} \quad \frac{7}{4} \quad -\frac{7}{4} \quad -\frac{9}{4}
\]

\[
\frac{3}{4} \quad \frac{7}{4} \quad -\frac{7}{4} \quad -\frac{9}{4}
\]

\[
\frac{3}{4} \quad \frac{7}{4} \quad -\frac{7}{4} \quad -\frac{9}{4}
\]

\[
\frac{3}{4} \quad \frac{7}{4} \quad -\frac{7}{4} \quad -\frac{9}{4}
\]
13. (a) Let \( y = x \cos x \)
\[ \ln y = \ln x \cos x + \ln y = x \ln x + \ln \cos x \]
\[ \frac{dy}{dx} = \left[ x \frac{1}{x} + \ln x + \frac{x \cos x}{\cos x} \right] dx \]
\[ \frac{dy}{dx} = \left[ 1 + \ln x - 3\tan 3x \right] (y) \]
\[ y' = x \cos 3x + (1 + \ln x - 3\tan 3x) \]

(i) Let \( y = 2 \ln(2x - 5) \)
Using product rule
\[ \frac{dy}{dx} = 2 \left( \frac{1}{2x-5} \right) + \left[ \ln(2x-5) \right] 2 \ln 2 \]

(ii) Let \( y = 3x^3 - 5x^2 + 2x \)
\[ \frac{dy}{dx} = 9x^2 - 10x + 2 \]

At \( (1, 1) \)
\[ y' = 1 \]
\[ (\text{the gradient of the tangent}) \]
\[ \frac{y' - 1}{x - 1} = \frac{y - x}{x - 1} \]
\[ y' = x \]

Using \( M_2 = -1 \)
\[ M_1 = 1 \]
\[ \frac{y' - 1}{x - 1} = -1 \]
\[ y' = -x + 1 \]
\[ x = 0 \]
\[ y = x + 2 \]

The equation of the normal is \[ y = x + 2 \]

14. L.H.S: \( \cos 4x = 2\cos^2 2x - 1 \)
\[ = 2\left[ 2\cos^2 x - 1 \right] - 1 \]
\[ = 8\cos^4 x - 8\cos^2 x + 1 \]
\[ \frac{8}{8} \sec^2 x - \frac{8}{\sec^2 x} + 1 \]
\[ \frac{8}{8} \sec x \cdot \sec x = \sec x \]

\[ \frac{8}{8} \sec^2 x = \sec^2 x \]

Dividing by \( \sec^2 x \) gives
\[ \frac{c^2 - b^2 + a^2m^2}{b^2 + c^2 - a^2} \]
\[ a^2m^2 - b^2m^2 + a^2m^2 = a^2 \]
\[ a^2m^2 = b^2m^2 \]

Equation of the tangent must be in the form \( y = mx + c \)
\[ y = mx \pm \sqrt{16m^2 + 9} \]
At \( (3, 3) \)
\[ 3 = 3m \pm \sqrt{16m^2 + 9} \]
\[ (3 + 3m)^2 = 16m^2 + 9 \]
\[ 9 + 18m + 9m^2 = 16m^2 + 9 \]
\[ 7m^2 - 18m = 0 \]
\[ m(7m - 18) = 0 \]